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QUANTITY-PRICE RELATIONSHIPS IN HEALTH INSURANCE

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One of the most basic, and yet most difficult, of all health insurance actuarial problems arises from the interdependence of utilization rates and the price paid by the benefited individual. The health insurance mechanism transfers some or all of the payment for health services from the individual directly served to the entity paying the health insurance premium. This transfer, by lowering the price to the individual, must be presumed to increase the utilization of health services.

This note is an attempt to explore the relationships between quantities of health services performed under conditions of no insurance, full insurance, and varying degrees of coinsurance. A theoretical framework will be attempted first, followed by a discussion of some of the possibilities for quantification.

The Induction Factor

If an individual uses a quantity q of health services, under conditions of no insurance, there must be some quantity q′ which he would use under conditions of full insurance.

If we define an induction factor α by

$$\frac{q'}{q} = 1 + \alpha$$

we can say that the quantity of services induced by the insurance mechanism is $q' - q = \alpha q$.

Very little is known about the magnitude of α, except that (1) it is presumably greater than zero, and (2) it would be expected to be a function of certain other variables, such as type of health service and the individual's income. The Office of the Actuary, Social Security Administration, used the concept of the induction factor α in its 1971 estimates[1] of the cost of various national health insurance proposals. For this purpose the various α's were assumed to be in the range .25 to .50. There is no theoretical reason, however, why α could not take a value as high as unity, or even higher.

The Uniform Induction Assumption

If we let price P represent the part of the total cost of health services that the individual pays directly, then P is defined only over the range $0 \leq P \leq 1$. The extremes $P = 0$ and $P = 1$ represent conditions of 100% insurance and no insurance, respectively. A 20% coinsurance is then represented by $P = .20$, a 25% coinsurance by $P = .25$.

Let Q_P represent quantity of services performed at price P. When $P = 0$, Q is at its maximum, which will be arbitrarily represented by $Q_0 = 1$. In accordance with the definition of the induction factor α,

$$Q_1 = \frac{1}{1 + \alpha}$$

Two points on the curve $Q = f(P)$ are now determined by definition; but the shape of the curve between the two points, and the value of α, remain to be investigated.

One approach with strong intuitive appeal to the shape of the curve is the assumption that the induction factor operates uniformly throughout its range. This uniform induction assumption was also employed by the Office of the Actuary in its estimates for various national health proposals. Each dollar transferred from an uninsured to a fully insured

[1] "Analysis of Health Insurance Proposals Introduced in the 92nd Congress"—Ways and Means Committee Print, U.S. G.P.O. (5270-1176), August 1971.

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status is assumed to induce α of additional services.

Stated mathematically, the uniform induction assumption states that a change in price (ΔP) causes a change in quantity (ΔQ) such that ΔQ is $-\alpha$ times the change in the dollar amount paid directly by the individual.

$$\Delta Q = -\alpha[(P + \Delta P)(Q + \Delta Q) - PQ]$$

$$\frac{\Delta Q}{\Delta P} = \frac{-\alpha(Q + \Delta Q)}{1 + \alpha P}$$

and taking the limit when ΔP approaches zero

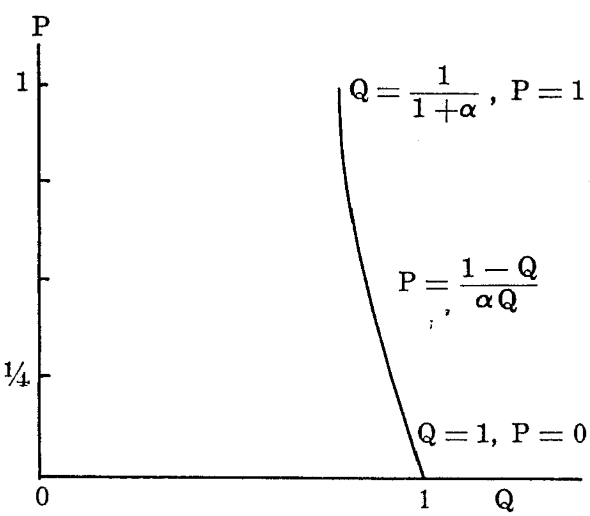
$$\frac{dQ}{dP} = \frac{-\alpha Q}{1 + \alpha P}$$

Solution of this differential equation, plus the establishment of Q_0 at unity, produces

$$Q = \frac{1}{1 + \alpha P} \qquad 0 \leq P \leq 1$$

$$\text{or } P = \frac{1 - Q}{\alpha Q} \qquad \frac{1}{1 + \alpha} \leq Q \leq 1$$

This curve, a portion of a hyperbola, has the characteristic appearance of price-quantity curves, except that price goes to zero, and as it does so quantity remains finite. In deference to the prevailing practice in the field of economics, it is graphed below as $P = f(Q)$, with the price P along the vertical axis, the quantity Q along the horizontal.



Quantification of Uniform Induction Model

The uniform induction assumption, leading to tne relationship $Q = \frac{1}{1 + \alpha P}$, enables us to calculate in terms of α the quantity of services used at any price P in the range $0 \leq P \leq 1$. An illustration of Q_0, $Q_{1/5}$, $Q_{1/4}$, and Q_1, based on various values of α, may be useful in visualizing the relationships.

α	Q_0	$Q_{1/5}$	$Q_{1/4}$	Q_1
0	1.00	1.00	1.00	1.00
0.1	1.00	0.98	0.98	0.91
0.2	1.00	0.96	0.95	0.83
0.3	1.00	0.94	0.93	0.77
0.4	1.00	0.93	0.91	0.71
0.5	1.00	0.91	0.89	0.67
0.7	1.00	0.88	0.85	0.59
1.0	1.00	0.83	0.80	0.50

It is of interest to note that the effect on utilization of a 20% coinsurance ranges from ¼ to ⅓ of the effect of eliminating all insurance.

The obvious way to calibrate the model, in order to obtain practical results, is a direct determination of α. If an experiment could be devised under which the utilization of a group of persons could be studied first under conditions of full insurance, then no insurance (or in the reverse order), then the α would be directly estimated for the particular group and for the particular kinds of services studied. The opportunities for such a study are very limited, however, and none seems to have been attempted.

Another possibility is to determine α by a direct determination of Q_P at some level of coinsurance. This essentially requires a study of a group of persons under conditions of full insurance $P = 0$, then at a definite level of coinsurance (or in the reverse order). As it happens there is a very interesting new study of this type,[2] based on 25% coinsurance. This study computes $Q_{1/4}$ for physicians' visits at about 0.75, for out-of-hospital ancillary services at about .80. The implied values of α are in the neighborhood of unity.

The Uniform Price Elasticity Assumption

The uniform induction model just described has at least one important competitor, the uniform price elasticity model.

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[2] "Effect of Coinsurance on Use of Physicians Services" Anne A. Scitovsky and Nelda M. Snyder, Social Security Bulletin, June 1972.

Price elasticity is defined as the ratio of the change in quantity (expressed as a percentage of quantity) to the change in price (expressed as a percentage of price). Thus the elasticity is the limit, as ΔP approaches zero, of $\frac{\Delta Q/Q}{\Delta P/P}$. Now if the elasticity ϵ is considered to be constant throughout the price range, and if its negative character is recognized by a minus sign in the formula (ϵ itself then being positive)

$$\frac{\Delta Q}{\Delta P} = -\epsilon \frac{Q}{P}$$
$$Q = kP^{-\epsilon}$$

or in linear-log form

$$\log Q = \log k - \epsilon \log P$$

For the particular health insurance problem in which we are interested, P has a range $0 \leq P \leq 1$, and in the formulae above Q increases without bound as P approaches zero. To avoid this difficulty we can imagine a time-price P_0, representative of the time and effort of getting medical attention, to be added to the cash-price P, to produce a total price P′. We can then think of P′ taking the range $P_0 \leq P' \leq 1 + P_0$. With this translation, and arbitrarily establishing Q at unity when the cash price P is zero, the formula above becomes

$$Q = \left(\frac{P + P_0}{P_0} \right)^{-\epsilon} \quad 0 \leq P \leq 1$$

$$\text{or } \log Q = -\epsilon [\log (P + P_0) - \log P_0]$$

Like the uniform induction model, the uniform elasticity model has an underlying parameter assumed to be constant throughout the range—in this case the elasticity ϵ. Unlike the uniform induction model, the uniform elasticity model has a second parameter, the time-price P_0. The value P_0 must of necessity be somewhat arbitrary; and the necessity for such a second parameter adds to the complexity of this model.

The graph of P against Q, for the uniform price elasticity model, tends to be somewhat more concave than the otherwise similar uniform induction model. In general, it attributes more effect to changes in coinsurance near $P = 0$, and less to that part of the curve near $P = 1$, than does the uniform induction model.

Quantification of Uniform Elasticity Model

Illustrations of Q_0, $Q_{1/5}$, $Q_{1/4}$, and Q_1, for various values of ϵ, and for P_0 set at .1 and .5, are shown below. The comparison with the similar display for the uniform induction model will point out the more rapid change, around $P = 0$, under the uniform elasticity model, particularly for small values of P_0.

ϵ	Q_0	$Q_{1/5}$	$Q_{1/4}$	Q_1
		$P_0 = .1$		
0	1.00	1.00	1.00	1.00
0.1	1.00	.90	.88	.79
0.2	1.00	.80	.78	.62
0.3	1.00	.72	.69	.49
0.4	1.00	.64	.61	.38
		$P_0 = .5$		
0	1.00	1.00	1.00	1.00
0.1	1.00	.97	.96	.90
0.2	1.00	.94	.92	.80
0.3	1.00	.90	.89	.72
0.4	1.00	.87	.85	.64

These illustrations also show that the uniform elasticity model is sensitive to the time-price P_0. As P_0 increases, this model more closely fits the uniform induction model.

The approach to establishing ϵ (and P_0) would be along the same lines as for the uniform induction model, though at least two points on the curve are needed.

Summary

Clearly, neither model can easily be "validated", nor is the choice between them at all obvious. Validation of either model, or a choice between them, involves the determination of several points on the graph of $Q = f(P)$. Determination of at least three points, in addition to the arbitrarily determined $Q_0 = 1$, would be most helpful—and should make actuaries and economists either more comfortable, or less so, with one or the other of the two models described.

The "calibration" of the uniform induction model can be accomplished with one additional point, since it contains a single parameter α. Two additional points are needed to calibrate the uniform elasticity model, with its two parameters ϵ and P_0.

A word of warning may be in order. Under either model the important parameters are probably variables. They would seem, by general reasoning, to be functions of income, with higher price sensitivity to be expected in low income families. They may well be functions of the type of health care—with different values for hospital than for physicians' services or out-of-hospital drugs. What is known about these relationships is clearly dwarfed by what is not known. Even so, the presentation of these two different mathematical models may be of help to persons working in this area.

The Social Security Administration is continuing its study of quantity-price relationships in the health area, and is expanding the models to recognize variation by family income.